AP Calculus AB

Unit 5 – Integration and Antiderivatives

AP Calculus AB – Worksheet 43 **Basic Integration and Antiderivatives**

Find all antiderivatives, $F(x)$, of each function.

1) $f(x)=4x^3$	2) $f(x)=\sin(2x)$	3) $f(x)=9\sec^2 x$
4) $f(y)=-\dfrac{12}{y^3}$	5) $f(x)=2e^x$	

Evaluate each indefinite integral.

6) $\int\left(7x^{20}-2x^5\right)dx$	7) $\int\left(9x^{-1/2}+9x^{1/2}\right)dx$	8) $\int\left(4t^{-2}+5t^6\right)dt$
9) $\int(3x+2)^2\,dx$	10) $\int\left(4x^{1/3}+3x^{-1/3}+9\right)dx$	11) $\int\dfrac{4x^5+10x^3}{x^2}dx$
12) $\int(\sin 3y+\cos 7y)\,dy$	13) $\int\left(-2\sec x\tan x-5\sec^2 x\right)dx$	14) $\int\left(e^{2x}+7x^{1/2}\right)dx$
15) $\int\dfrac{6+u}{u}du$	16) $\int\dfrac{16t^8+6}{t}dt$	

Answers

1) $F(x)=x^4+C$	2) $F(x)=-\dfrac{1}{2}\cos(2x)+C$	3) $F(x)=9\tan x+C$
4) $F(y)=6y^{-2}+C$	5) $F(x)=2e^x+C$	6) $\dfrac{1}{3}x^{21}-\dfrac{1}{3}x^6+C$
7) $18x^{1/2}+6x^{3/2}+C$	8) $-4t^{-1}+\dfrac{5}{7}t^7+C$	9) $\dfrac{1}{9}(3x+2)^3+C$
10) $3x^{4/3}+\dfrac{9}{2}x^{2/3}+9x+C$	11) x^4+5x^2+C	12) $-\dfrac{1}{3}\cos 3y+\dfrac{1}{7}\sin 7y+C$
13) $-2\sec x-5\tan x+C$	14) $\dfrac{1}{2}e^{2x}+\dfrac{14}{3}x^{3/2}+C$	15) $6\ln\lvert u\rvert+u+C$
16) $2t^8+6\ln\lvert t\rvert+C$		

AP Calculus AB – Worksheet 44 **Box Method/ "Most Complicated" Rule**

Problems:

1. $\int (x+2)^4 \, dx$	2. $\int (1-x)^9 \, dx$	3. $\int \frac{dx}{(4-x)^2}$	4. $\int \frac{dx}{x+2}$
5. $\int \frac{dx}{2x-1}$	6. $\int \frac{dx}{(3x+2)^2}$	7. $\int x(x^2-1)^4 \, dx$	8. $\int \frac{x}{1+x^2} \, dx$
9. $\int \frac{x}{\sqrt{1-x^2}} \, dx$	10. $\int x\sqrt{x^2-1} \, dx$	11. $\int xe^{x^2} \, dx$	12. $\int x \sin x^2 \, dx$
13. $\int e^x \cos e^x \, dx$	14. $\int \cos x(e^{\sin x}) \, dx$	15. $\int (2x-1)^4 \, dx$	16. $\int \frac{dx}{1-x}$
17. $\int \frac{x}{1-x^2} \, dx$	18. $\int x(x^2+1)^9 \, dx$	19. $\int x\sqrt{1-x^2} \, dx$	20. $\int \frac{x}{\sqrt{x^2+4}} \, dx$
21. $\int \frac{x^2}{x^3+1} \, dx$	22. $\int x^2\sqrt{x^3+1} \, dx$	23. $\int \frac{\cos x}{1+\sin x} \, dx$	24. $\int \frac{e^x}{1+e^x} \, dx$

Answers:

1. $\frac{(x+2)^5}{5}+C$	2. $\frac{-(1-x)^{10}}{10}+C$	3. $\frac{1}{4-x}+C$	4. $\ln\lvert x+2\rvert+C$
5. $\frac{1}{2}\ln\lvert 2x-1\rvert+C$	6. $-\frac{1}{3(3x+2)}+C$	7. $\frac{(x^2-1)^5}{10}+C$	8. $\frac{1}{2}\ln(1+x^2)+C$
9. $-\sqrt{1-x^2}+C$	10. $\frac{\sqrt{(x^2-1)^3}}{3}+C$	11. $\frac{e^{x^2}}{2}+C$	12. $\frac{-\cos x^2}{2}+C$
13. $\sin e^x+C$	14. $e^{\sin x}+C$	15. $\frac{(2x-1)^5}{10}+C$	16. $-\ln\lvert 1-x\rvert+C$
17. $-\frac{1}{2}\ln\lvert 1-x^2\rvert+C$	18. $\frac{(x^2+1)^{10}}{20}+C$	19. $-\frac{\sqrt{(1-x^2)^3}}{3}+C$	20. $\sqrt{x^2+4}+C$
21. $\frac{1}{3}\ln\lvert x^3+1\rvert+C$	22. $\frac{2\sqrt{(x^3+1)^3}}{9}+C$	23. $\ln(1+\sin x)+C$	24. $\ln(1+e^x)+C$

AP Calculus AB – Worksheet 45 **"Most Complicated" Rule 2**

Evaluate each indefinite integral.

1) $\int (4-3s)^{1/2}\,ds$	2) $\int (9t-8)^{-1/2}\,dt$
3) $\int x\left(9+x^2\right)^{1/9}\,dx$	4) $\int \cos(8z-9)\,dz$
5) $\int 2x\left(x^2+6\right)^3\,dx$	6) $\int x^2 e^{x^3}\,dx$
7) $\int 3x^4\left(1-12x^5\right)^{-1/2}\,dx$	8) $\int \left(x^2+x\right)^{12}(2x+1)\,dx$
9) $\int \dfrac{dx}{2x+1}$	10) $\int \dfrac{e^{5x}}{e^{5x}+9}\,dx$
11) $\int \sec^2(6x+1)\,dx$	12) $\int \dfrac{x^7}{\left(9-x^8\right)^2}\,dx$
13) $\int \left(3x^4-4x^{-5}+\sec^2 x\right)dx$	

Answers

1) $-\dfrac{2}{9}(4-3s)^{3/2}+C$	2) $\dfrac{2}{9}(9t-8)^{1/2}+C$	3) $\dfrac{9}{20}\left(9+x^2\right)^{10/9}+C$
4) $\dfrac{1}{8}\sin(8z-9)+C$	5) $\dfrac{1}{4}\left(x^2+6\right)^4+C$	6) $\dfrac{1}{3}e^{x^3}+C$
7) $-\dfrac{1}{10}\left(1-12x^5\right)^{1/2}+C$	8) $\dfrac{1}{13}\left(x^2+x\right)^{13}+C$	9) $\dfrac{1}{2}\ln\lvert 2x+1\rvert+C$
10) $\dfrac{1}{5}\ln\left\lvert e^{5x}+9\right\rvert+C$	11) $\dfrac{1}{6}\tan(6x+1)+C$	12) $\dfrac{1}{8}\left(9-x^8\right)^{-1}+C$
13) $\dfrac{3}{5}x^5+x^{-4}+\tan x+C$		

AP Calculus AB – Worksheet 46 **Integration of Inverse Trigonometric Functions**

Evaluate each integral.

1. $\int \frac{dx}{x^2+9}$	2. $\int \frac{9r^2}{\sqrt{1-r^3}}\,dr$	3. $\int \cos(3z+4)\,dz$	4. $\int \frac{dx}{\sin^2 3x}$
5. $\int \frac{6\cos t}{(2+\sin t)^2}\,dt$	6. $\int \frac{x}{x^2+1}\,dx$	7. $\int \frac{40}{x^2+25}\,dx$	8. $\int \frac{dx}{\sqrt{1-4x^2}}$
9. $\int \frac{2y}{4y^2-1}dy$	10. $\int \frac{dx}{\sqrt{9-x^2}}$		

Answers:

1. $\left(\frac{1}{3}\right)\tan^{-1}\left(\frac{x}{3}\right)+C$	2. $-6\sqrt{1-r^3}+C$	3. $\frac{1}{3}\sin(3z+4)+C$	4. $-\frac{1}{3}\cot(3x)+C$
5. $-\frac{6}{2+\sin t}+C$	6. $\frac{1}{2}\ln(x^2+1)+C$	7. $8\tan^{-1}\left(\frac{x}{5}\right)+C$	8. $\sin^{-1}(2x)+C$
9. $\frac{1}{4}\ln\lvert 4y^2-1\rvert+C$	10. $\sin^{-1}\left(\frac{x}{3}\right)+C$		

AP Calculus AB – Worksheet 47 **The Definite Integral**

Evaluate each definite integral.

1) $\int_0^1 \left(5x+\sqrt{x}\right)dx$	2) $\int_{\pi/3}^{3\pi/4} \sin x dx$
3) $\int_{-3}^4 \left(2x^3+9\right)dx$	4) $\int_1^8 x^{-5/3}dx$
5) $\int_{\frac{1}{2}}^4 x^{-2}dx$	6) $\int_0^{\pi/3} 7\sec^2 x dx$
7) $\int_1^6 3t^{-1}dt$	8) $\int_0^{\pi/42} \cos 7x dx$
9) $\int_0^5 6e^{3x}dx$	10) $\int_0^1 3x^2\left(2-x^3\right)dx$
11) $\int_0^2 16x^3e^{x^4}dx$	12) $\int_0^2 \left(x^3+5\right)^{-1}x^2dx$
13) $\int_0^9 \frac{1}{4+3\sin x}dx$	

Answers

14) $\frac{19}{6}$	15) $\frac{\sqrt{2}+1}{2}$	16) $[128+36]-\left[\frac{81}{2}-27\right]$
17) $\frac{9}{8}$	18) $\frac{7}{4}$	19) $7\sqrt{3}$
20) $3\ln 6$	21) $\frac{1}{14}$	22) $2e^{15}-2$
23) $\frac{3}{2}$	24) $4e^{16}-4$	25) $\frac{1}{3}\ln\left(\frac{13}{5}\right)$
26) 2.829		

AP Calculus AB - Worksheet 48 **Integrate with initial values**

Find the general solution to the exact differential equation.

1. $\frac{dy}{dx} = 5x^4 - \sec^2 x$	2. $\frac{dy}{dx} = \sin x - e^{-x} + 8x^3$	3. $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$, $(x > 0)$	4. $\frac{dy}{dt} = 3t^2 \cos\left(t^3\right)$

Find the particular solution to the differential equation.

5. $\frac{dy}{dx} = 3\sin x$; $y = 2$ when $x = 0$	6. $\frac{dy}{dx} = 2e^x - \cos x$; $y = 3$ when $x = 0$	7. $f'(x) = 7x^6 - 3x^2 + 5$; $f(1) = 1$	8. $y' = 10x^9 + 5x^4 - 2x + 4$ $y = 6$ when $x = 1$
9. $f'(x) = -\frac{1}{x^2} - \frac{3}{x^4} + 12$ $f(1) = 3$	10. $\frac{dy}{dx} = 5\sec^2 x - \frac{3}{2}\sqrt{x}$; $y\big\|_{x=0} = 7$	11. $\frac{dx}{dt} = \frac{1}{t} - \frac{1}{t^2} + 6$; $x = 0$ when $t = 1$	12. $\frac{dv}{dt} = 4\sec t \tan t + e^t + 6t$; $v = 5$ when $t = 0$
13. $\frac{d^2y}{dx^2} = 24x^2 - 10$. When $x = 1$, $\frac{dy}{dx} = 3$ and $y = 5$.		14. $f''(x) = \cos x - \sin x$. $f'(0) = 2$ and $f(0) = 0$	

Answers:

1. $y = x^5 - \tan x + C$	2. $y = -\cos x + e^{-x} + 2x^4 + C$	3. $y = \ln x + x^{-1} + C$	4. $y = \sin\left(t^3\right) + C$
5. $y = -3\cos x + 5$	6. $y = 2e^x - \sin x + 1$	7. $f(x) = x^7 - x^3 + 5x - 4$;	8. $y = x^{10} + x^5 - x^2 + 4x + 1$;
9. $f(x) = x^{-1} + x^{-3} + 12x - 11$	10. $y = 5\tan x - x^{\frac{3}{2}} + 7$; $(0 < x < \frac{\pi}{2})$	11. $x = \ln t + t^{-1} + 6t - 7$; $(t > 0)$	12. $v = 4\sec t + e^t + 3t^2$; $\left(-\frac{\pi}{2} < t < \frac{\pi}{2}\right)$ Note that $C = 0$.
13. $y = 2x^4 - 5x^2 + 5x + 3$		14. $f(x) = -\cos x + \sin x + x + 1$	

AP Calculus AB - Worksheet 49 **Integration Review 1**

1) $\int_1^2 \left(2x-3x^2+4x^3\right)dx$	2 $\int_0^1 x^6\left(4x^2+x-5\right)dx =$	3) $\int_3^4 \frac{3x}{x^2-8}dx =$
4) $\int 7e^{-5x}dx =$	5) $\int \cos^5 x \sin x\, dx =$	6) $\int_{\ln 2}^{\ln 9} 4e^x dx =$
7) $\int_4^{25} \frac{e^{\sqrt{x}}}{\sqrt{x}}dx =$	8) $\int_{\frac{\sqrt{3}}{3}}^1 \frac{dx}{1+x^2} =$	9) $\int \cos(3-2x)dx =$
10) $\int \frac{x}{\sqrt{9-x^2}}dx$	11) $\int \frac{3x}{\left(x^2+1\right)^4}dx =$	12) $\int x\cos x^2\ dx$
13) $\int \cos^3 2u \sin 2u du =$	14) Find the antiderivative of $\frac{2\cos x}{\sin^2 x}$	15) Given that $f'(x)=\frac{1}{2}x^2+\frac{3}{4}x$ and $f(1)=2$. Find $f(x)$.
16) If $f'(x)=5x^4-2x$ and if $f(1)=6$, then $f(2)=?$	17) $\lim_{x\to 4}\frac{x^2-4}{x^2-16}$	18) $\lim_{x\to 6}\frac{x-6}{x^2-36}$
19) $f''(x)=x; f'(2)=0; f(0)=4$, find $f(1)$	20) $\int_0^{\pi} \sec x \tan x dx$	21) $\int_0^1 \frac{e^x}{\left(3-e^x\right)^2}dx$

Answers:

1) 11	2) $-\frac{73}{504}$	3) $\frac{3}{2}\ln 8$	4) $-\frac{7}{5}e^{-5x}+C$	5) $-\frac{1}{6}\cos^6 x+C$	6) 28
7) $2\left(e^5-e^2\right)$	8) $\frac{\pi}{12}$	9) $-\frac{1}{2}\sin(3-2x)+C$	10) $-\sqrt{9-x^2}+C$	11) $-\frac{1}{2\left(x^2+1\right)^3}+C$	12) $\frac{1}{2}\sin x^2+C$
13) $-\frac{1}{8}\cos^4 2u+C$	14) $-\frac{2}{\sin x}+C$	15) $f(x)=\frac{x^3}{6}+\frac{3}{8}x^2+\frac{35}{24}$	16) $f(2)=34$	17) DNE	18) $\frac{1}{12}$
19) $\frac{13}{6}$	20) –2	21) $\frac{e-1}{2(3-e)}$			

AP Calculus AB – Worksheet 50 **Integration Review 2**

1. $\int_0^1 \frac{x}{x^2+1}\,dx$	2. $\int_0^{\frac{\pi}{2}} \cos^2 x \sin x\,dx$
3. $\int_1^2 (3x-2)^3\,dx$	4. $\int_{-1}^0 e^{-x}\,dx$
5. $\int_1^3 \frac{6x-2}{x}dx$	6. $\lim_{x\to 5} \frac{2x^2-9x-5}{x-5}$
7. $f'(x)=3x^2+4$ and $f(2)=14$, find $f(1)$	8. $\int \frac{1}{\sqrt{16-x^2}}\,dx$
9. $\int_0^1 x^2(15-4x)dx$	10. Find $\frac{dy}{dx}$ for $3y^2-4x=7xy$

Answers:

1) $\ln\sqrt{2}$	2) $\frac{1}{3}$	3) $\frac{85}{4}$	4) $e-1$
5) $12-\ln 9$	6) 11	7)) 3	8) $\arcsin\left(\frac{x}{4}\right)+C$
9) 4	10) $\frac{dy}{dx}=\frac{4+7y}{6y-7x}$		